Title: Fractals

Topics: symmetry, patterns in nature, biomimicry

Related Disciplines: mathematics, biology

Objectives:

A. Learn about how fractals are made.
B. Think about the mathematical processes that play out in nature.
C. Create a hands-on art project.

Lesson:

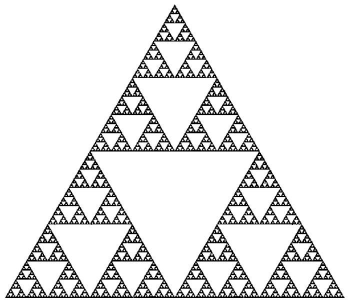
A. Introduction (20 minutes)

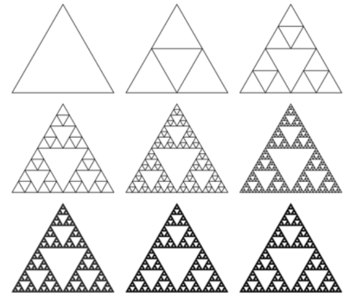
Figure 1: Romanesco

Fractals are sets in mathematics based on repeated processes at different scales. Why do we care? Fractals are a part of nature, geometry, algebra, and science and are beautiful in their complexity. Fractals come in many types: branching, spiral, algebraic, and geometric. Branching and spiral fractals can be found in nature in many different forms. Branching is seen in trees, lungs, snowflakes, lightning, and rivers. Spirals are visible in natural forms such as hurricanes, shells, liquid motion, galaxies, and most easily visible in plants like flowers, cacti, and Romanesco (Figure 1). Branching fractals emerge in a linear fashion. Spirals begin at a point and expand out in a circular motion. A cool demonstration of branching fractals can be found at: http://fractalfoundation.org/resources/what-are-fractals/. Algebraic fractals are based on surprisingly simple equations. Most famous of these equations is the Mandelbrot Set (Figure 2), a plot made from the equation:

$$z_{n+1} = z_n^2 + C$$

The most famous of the geometric fractals is the Sierpinski Triangle:





As seen in the diagram above, this complex fractal is created by starting with a single triangle, then forming another inside one quarter the size, then three more, each one quarter the size, then 9, 27, 81, all just one quarter the size of the triangle drawn in the previous step. The geometric project we will complete later on in class is the three-dimensional equivalent of this. This fractal is highly biomimetic because almost all fractals seen in nature arise from one large structure like the trunk of a tree, or the basic geometry of a snowflake, and repeat a basic pattern to form the overall structure. The functionality is seen in the efficient structure, and beauty is found in the symmetry of natural forms.

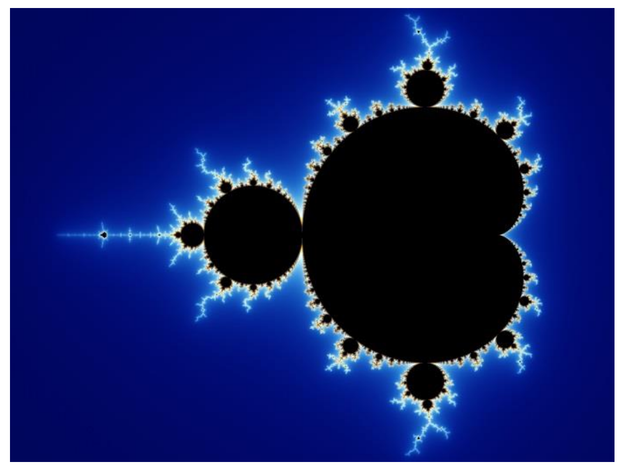


Figure 2: Mandelbrot Set

We have only recently begun attempting to use fractals in engineering and medicine. Computer chips are one of the most common uses in daily life, where circuit boards are designed in the shape of repeating squares and rectangles.

<u>B. Class Project (60 minutes)</u>

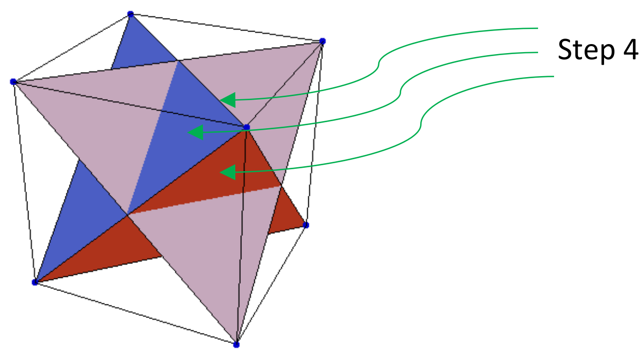
For the art project itself, we are making tetrahedral fractals, using a similar procedure to forming the Sierpinski Triangle.

Here are step-by-step instructions on how to build fractala. These instructions do not need to be read aloud to the class. They can be summarized verbally or printed for groups of older students to read and follow in small groups.

1. Have an initial tetrahedron (triangle pyramid) made to demonstrate how students should make them.
    a. To make a tetrahedron, cut out four triangles with equal side lengths of 12 inches from poster paper (colored paper is optional).

b. Attach three of the triangles to the sides of a central one. Create the shape of a pyramid by attaching the three outer triangles together, using glue or tape.

2. Have students cut out their own tetrahedrons.
3. After these tetrahedrons are built, have students cut out 16 triangles with side lengths of 6 inches and build four smaller tetrahedrons.
    a. Each tetrahedron will go on one of the sides of the initial tetrahedron, with the corner of each triangle at the middle of an edge on the initial triangle.
    b. If done correctly, it should look like:



C. Conclusion (10 minutes)

Now that each group has completed or tried to complete a “fractala,” reflect on what was learned through today’s activity. Below is a list of questions that may be incorporated into the discussion:

1. What did you learn about the process of fractals forming?
2. Was the process easier to complete the longer you had been doing it? Do you think nature takes this into account?
3. Why does nature value the repetitive shapes of fractals?
4. Calculate the volume and/or surface area of the shape after each step. What can be seen?
5. How are nature and math connected based on what we’ve seen today?
6. How are math and art connected?

Homework:

Find an artist who utilizes math in his or her artwork and explain how the use of mathematics impacts the piece.

Further Reading:

http://fractalfoundation.org/resources/what-are-fractals/

http://mathworld.wolfram.com/SierpinskiSieve.html

http://mathworld.wolfram.com/MandelbrotSet.html